Quantum chaos and quantum gravity

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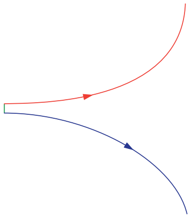
Quantum chaos?

What is **Quantum** Chaos ?

Butterfly effect

- In classical mechanics strong chaos is synonymous with:
- Sensitive dependence on initial conditions.
- $v(0), v(0) + \delta v(0)$ two nearby points in phase space:



- $|\delta v(t)| \sim e^{\lambda_L t} |\delta v(0)|$.
- Here λ_L is a Lyapunov exponent.
- What is the "quantum butterfly effect"?

Quantum diagnostics

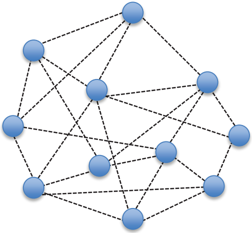
- First idea – two nearby states in Hilbert space:
- $|\chi\rangle, |\chi'\rangle, \quad \| \, |\chi\rangle - |\chi'\rangle \, \| = \epsilon.$
- Then evolve in time.
- $\| \, e^{-iHt}(|\chi\rangle - |\chi'\rangle) \, \| = \epsilon$
- Distance between quantum states does not change with time under unitary evolution.
- Need something better....

Quantum diagnostics

- General picture:
- Consider $W(t) = e^{iHt} W(0) e^{-iHt}$.
- Forward time evolution, apply simple operator, then backward time evolution.
- Chaos causes a lack of cancellation, Even if $W(0)$ is simple, $W(t)$ becomes a complicated operator.

Operator growth

A model: a set of N qubits, connected in groups of four.



$$H = \sum_{ijkl} J_{ijkl} \sigma_i \sigma_j \sigma_k \sigma_l$$

(Here σ_i is at site i and is short for σ_i^x, σ_i^y and σ_i^z. Take the J_{ijkl} to be randomly distributed, and different for the various x, y, z choices. In the SYK model the qubits are replaced by Majorana fermions.)

Operator growth, contd.

$$\sigma_1(t) = e^{iHt}\sigma_1 e^{-iHt}$$
$$\sim \sigma_1 + t[H,\sigma_1] + t^2[H,[H,\sigma_1]] + t^3[H,[H,[H,\sigma_1]]] + \ldots$$

Remember $[\sigma,\sigma] \sim \sigma$ so

$$\sim \sigma_1 + t\sum_{jkl}\sigma_1\sigma_j\sigma_k\sigma_l + t^2\sum\sigma_1\sigma\ldots\sigma + t^3\sum\sigma_1\sigma\sigma\ldots\sigma\sigma + \ldots$$

- The operator "grows" in time, exponentially (each σ can "connect" to three others).
- The exponential growth of the size – the average number of σ's in each string – defines a "quantum Lyapunov exponent".

Gravitational analog

- In certain large N quantum systems, strongly coupled quantum behavior is "dual" to classical gravitational behavior (gauge/gravity duality, AdS/CFT).
- Finite temperature systems are dual to black holes.
- What is the classical gravitational analog of quantum Lyapunov behavior?

Chaos in black holes

- Set up a fine tuned initial condition and tests its sensitivity to perturbation:

- A photon starting just outside the horizon, tuned to stay close for a long time t and finally depart.
- Requires starting exponentially close to the horizon, because the distance from the horizon increases exponentially in time.
- Requires an exponentially large amount of energy, which "redshifts" away.

Chaos in black holes, contd.

- Now perturb this initial condition a bit by dropping a particle into the black hole.

- The black hole becomes more massive, the horizon grows and swallows the photon.

Chaos in black holes, contd.

- The photon is trapped behind the horizon so it inevitably hits the singularity, a dramatically different outcome than before.
- If t is large the photon starts exponentially close to the horizon, so an exponentially small mass increase is enough to cause this.
- This exponential sensitivity is a signal of Lyapunov behavior.

Chaos bound

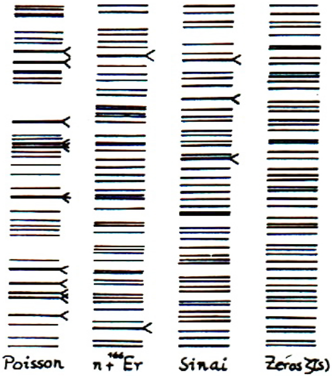
- The classical gravitational calculation, when translated to the quantum system, gives
- $\lambda_L = 2\pi k_B T/\hbar$.
- This turns out to be a universal upper bound on the rate of development of quantum chaos in systems with many degrees of freedom.
- "Black holes are the fastest scramblers."

Quantum gravity?

- This is fine as far as it goes, but the gravitational phenomena involved are basically classical.
- Can we learn anything about quantum gravity from quantum chaos?

Energy levels in quantum chaotic systems

- Quantized energy levels in quantum chaotic systems show a distinctive pattern:



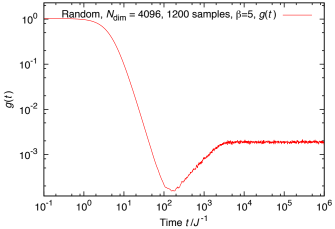
- In chaotic systems neighboring energy levels repel: “short range level repulsion.”
- The “gas” of energy levels is hard to compress: “long range spectral rigidity.”

Random matrix statistics

- The energy levels of quantum chaotic systems are widely believed to have the same statistical properties as those of **random matrices**.
- A remarkable example of universality. Independent of dimension, locality. Only weakly dependent on symmetry.
- Should reflect a universal phenomenon in quantum gravity as well.
- A nonperturbative phenomenon.

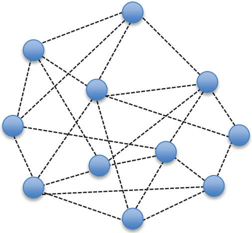
Spectral form factor

- The spectral form factor is a quantitative diagnostic of random matrix statistics.
$$\sum_{n,m} e^{i(E_n - E_m)t}$$
- The Fourier transform of the energy difference distribution.



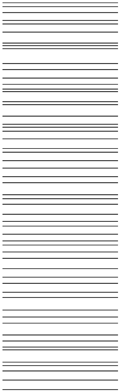
- The "slope" depends on the detailed ensemble.
- The "ramp" is a universal signature of long range spectral rigidity.
- The "plateau" is a universal signature of short range energy level repulsion. Sets in at a time corresponding to the inverse level spacing $\sim e^S$.

Study structure of energy eigenvalues of a (model) black hole.



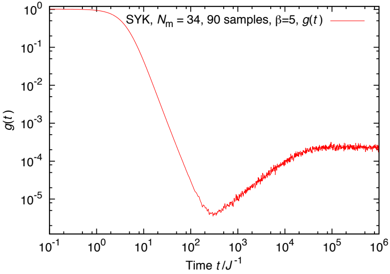
Sachdev-Ye-Kitaev (SYK) model

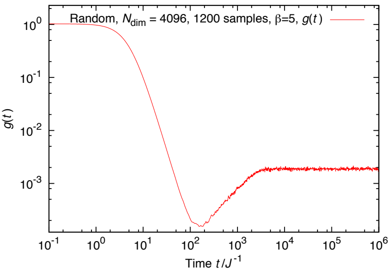
$$H_{\mathrm{SYK}} = \sum_{abcd}^{N} J_{abcd} \psi_a \psi_b \psi_b \psi_d$$



Spectral form factor

- Compute the spectral form factor for the SYK model
- (diagonalize 90 samples of 64K $\times$ 64K matrices)





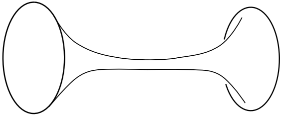
- Quantitative agreement with random matrix statistics.
- Spectral rigidity extends over an exponentially large number of energy levels.

What does it mean in quantum gravity?

- Generic black holes are chaotic so this pattern should be generic.
- SYK has a low energy sector dual to a kind of 2D gravity (Jackiw-Teitelboim gravity) .
- Find the explanation there.

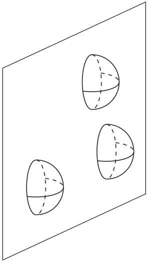
The ramp

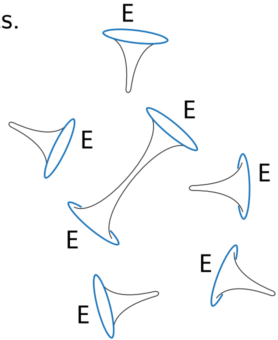
- It turns out that the "ramp" is due to another saddle point of gravity, the "cylinder" or "annulus." A kind of Euclidean wormhole.



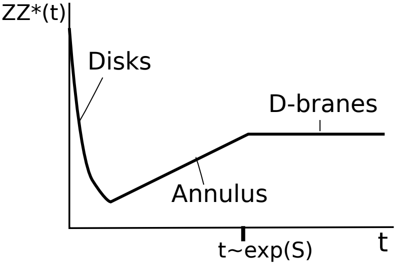
The plateau

- 2D gravity on various topologies "looks" like a string theory, where here perturbative string splitting and joining is the nonperturbative joining and splitting of closed 2D "baby universes."
- It turns out that the plateau is due to a spacetime analog of "D-branes." These are nonperturbative effects in string theory. Here they are "doubly" nonperturbative effects.





Summary



What does it mean?

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